at 0°C. The electron-energy distribution functions for Ar-H_2 mixtures with $E/p=0.5$ and $E/p=1.0$ at 0°C are plotted in Figs. 3 and 4, respectively. The electron average energy as a function of the carbon dioxide density to argon density ratio for $E/p=0.5$ and $E/p=1.0$ is plotted in Fig. 5. The electron average energy as a function of the molecular hydrogen density to argon density ratio for $E/p=0.5$ and $E/p=1.0$ is plotted in Fig. 6.

The energy distribution function for electrons in pure argon at $E/p=1.0$ given in Figs. 2 and 4 is almost identical with that published by Barbiere.[5] The effect of the addition of small amounts of carbon dioxide

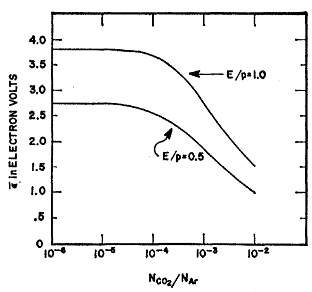

FIG. 5. Electron average energy versus ratio of carbon dioxide density to argon density for $E/p=0.5$ V/cm/mm-Hg and $E/p=1.0$ V/cm/mm-Hg at 0°C.

[5] D. Barbiere, Phys. Rev. 84, 653 (1951).

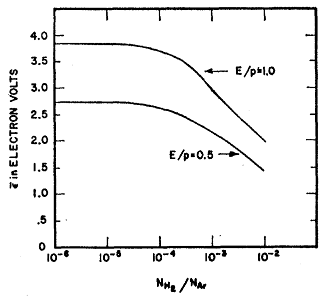

FIG. 6. Electron average energy versus ratio of molecular hydrogen density to argon density for $E/p=0.5$ V/cm/mm-Hg and $E/p=1.0$ V/cm/mm-Hg at 0°C.

or molecular hydrogen to the argon is to lower the electron energies. From a physical point of view, the electron energies are lowered due to the relatively large fractional energy loss per collision that an electron suffers in an inelastic collision with carbon dioxide or with hydrogen. This lowering of the electron energies is more pronounced in the Ar-CO_2 mixtures than in the Ar-H_2 mixtures because carbon dioxide can absorb a greater fraction of an electron's energy in a collision than can molecular hydrogen. It can be seen from the figures that one part of carbon dioxide or of molecular hydrogen in 10 000 parts of argon is sufficient to alter appreciably the electron-energy distribution function and the electron average energy from the values they would have in pure argon.

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Classical Impulse Approximation for Inelastic Electron-Atom Collisions

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Analytic expressions for the ionization and excitation cross sections of atoms by electrons are derived using the classical impulse approximation, i.e., by considering only the Coulomb interaction between the incident electron and one bound electron. The results obtained are slightly simpler and more self-consistent than those obtained in an earlier calculation by Gryzinski. The cross sections are found to be roughly as good as those obtained by the Born approximation except in the high-energy limit. The apparent superiority of Gryzinski's theory to quantum approximations arises from a subsidiary approximation made in averaging the cross section over the initial angular distribution rather than from the kinematic description of the bound electrons or the nature of the impulse approximation itself. The Coulomb cross section for transfer of energy ΔE between two particles of equal mass m, charge e, initial kinetic energies E_1 and E_2, relative velocity V, with an isotropic initial angular distribution is found to be

$$Vd\sigma/d(\Delta E)=2^{1/2}\pi e^4|\Delta E|^{-2}(mE_1E_2)^{-1/2}(\mathcal{E}^{1/2}+\tfrac{2}{3}\mathcal{E}^{3/2}/|\Delta E|),$$

where $\mathcal{E}$ is the smallest of the four initial and final kinetic energies. For single ionization this cross section is found to increase as the 3/2 power of the excess energy above threshold, reach a maximum at about $2\frac{1}{2}$ times the threshold energy, and decrease as E^{-1} at high energies. For hydrogenic atoms in any state the cross section goes to 5/3 the classical Thomson ionization cross section in the high-energy limit.

INTRODUCTION

UNTIL recently there has been no acceptable treatment of inelastic electron-atom collisions by the classical impulse approximation—that is, by calculating the cross sections for energy transfer in binary electron-electron collisions, neglecting the field of the nucleus and other bound electrons. Some time ago J. J. Thomson[1] treated inelastic electron-atom collisions by considering the Coulomb scattering of the

[1] J. J. Thomson, Phil. Mag. 23, 419 (1912).

incident electron by an atomic electron at rest. The neglect of the motion of the bound electron is certainly not justified at low or intermediate incident energies and, surprisingly, yields too small a value for the classical impulse approximation cross section in the high-energy limit (see below).

Gryzinski[2] has greatly improved the status of this approximation by allowing for the motion of the bound electrons. He calculates classical cross sections for an arbitrary energy transfer to a bound electron from an incident electron or heavy particle. From these, ionization cross sections can be deduced by imposing a correspondence between the final kinetic energy of the target electron and the energy levels of the atom. Gryzinski's results also yield a quantum impulse approximation to these inelastic cross sections insofar as the classical and quantum cross sections for Coulomb scattering are the same in the absence of relativistic or exchange effects.[3] Thus, the significant approximation made is not that of classical mechanics but rather that of neglecting the effects of any third bodies (the nucleus, other atomic electrons) on the motions of the incident particle and target electron.[4] Gryzinski's cross sections are in remarkably good agreement with experimental data for a wide variety of inelastic processes. The results appear to indicate that the properly calculated impulse approximation is superior not only to the earlier classical theory, but also to many first- and second-order perturbation theories of inelastic electron-atom collisions.

What has not been pointed out is that a subsidiary approximation made by Gryzinski in averaging over the initial angular distribution is responsible for the fact that his cross sections are in any better agreement with experiment than the Born approximation. This second approximation, rather than simplifying the forms of the cross sections, actually complicates them; and while it does improve the results, it enters in an arbitrary fashion which removes much of the self-consistency of the calculation (e.g., the cross sections do not behave properly under time reversal).

In this paper we derive the "exact" classical impulse approximation and obtain a number of simple analytic cross sections for ionization and excitation of atoms by electrons. These cross sections are generally the same at threshold and in the high-energy limit as those obtained by Gryzinski but lie somewhat above them in the intermediate energy domain. The ionization cross section duplicates the Born approximation at low energies and falls below it at high energies, whereas the excitation cross sections fall below it at all energies.

[2] M. Gryzinski, Phys. Rev. **115**, 374 (1959).

[3] R. Akerib and S. Borowitz, Phys. Rev. **122**, 1177 (1961); the calculations presented in this reference probably are not reliable [S. Borowitz (private communication)]. See also W. F. Ford, Bull. Am. Phys. Soc. **8**, 435 (1963).

[4] Noted by M. J. Seaton, review paper presented at the Third International Conference on the Physics of Electronic and Atomic Collisions, July 1963 (to be published).

The form of the classical cross sections obtained below thus shows that allowance for the motion of the bound electrons does improve the agreement with experimental data over that obtained with the Thomson formula. These cross sections do not, however, constitute any significant improvement over quantum approximations except in their greater simplicity.

DESCRIPTION OF MODEL

The model for the classical impulse approximation for electron-atom collisions consists of neglecting all terms in the Hamiltonian except the kinetic energies of the target electron E_1 and the incident electron E_2 and the interaction between them e^2/r_{12}. We are left with the problem of calculating the cross section for the scattering of two electrons in the laboratory frame of reference. Rather than the usual differential cross section, however, we seek the cross section per unit energy transfer ΔE.[5] Later we shall interpret collisions in which

$$\Delta E \equiv E_2' - E_2 \leq -U, \tag{1}$$

where E_2' is the final kinetic energy of the incident electron and U is the ionization potential of the target electron, to result in ionization of the atom. Excitation of the state n may be defined analogously, but with somewhat less confidence, to occur when[2]

$$U_n \leq -\Delta E \leq U_{n+1}, \tag{2}$$

where U_n is the total energy of the level n with respect to the total energy of the initial configuration of the atom. Clearly, this approach is only applicable to states resulting from excitation (or de-excitation) of a single electron in the initial configuration.

The total cross section σ for the scattering of two particles with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ may be obtained in the form

$$V\sigma(\mathbf{v}_1,\mathbf{v}_2) = |(\mathbf{v}_2 - \mathbf{v}_1)\cdot\hat{n}| \int P(\mathbf{s})d^2s, \tag{3}$$

where $V \equiv |\mathbf{v}_1 - \mathbf{v}_2|$, $P(\mathbf{s})$ is the probability for a collision at a separation of the velocities vectors in configuration space of $\mathbf{s}$, and the integration is performed over a plane in configuration space whose normal is $\hat{n}$. The cross section for transfer of energy between ΔE and $\Delta E + d(\Delta E)$ is given by

$$Vd\sigma/d(\Delta E) = |(\mathbf{v}_2 - \mathbf{v}_1)\cdot\hat{n}| \int P(\mathbf{s})\delta[\Delta E(\mathbf{s})]d^2s. \tag{4}$$

Because the Coulomb field has infinite range and because we are treating the collision classically we have $P(\mathbf{s}) = 1$ for all $\mathbf{s}$.

[5] The calculation is carried out in the laboratory frame only because ΔE is not an invariant under transformation of the reference system.

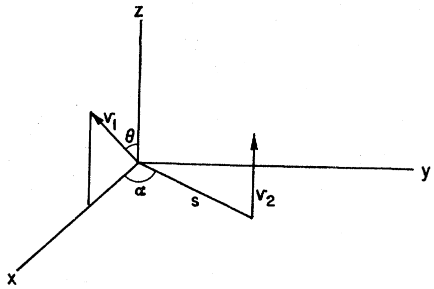

FIG. 1. Coordinates for calculation of the Coulomb scattering cross section per unit energy transfer in the laboratory frame.

CALCULATION OF THE COULOMB CROSS SECTION

The integration of Eq. (4) can be performed over any plane in configuration space but it is convenient to choose one normal to $\mathbf{v}_2$. Then, as shown in Fig. 1. we integrate over the x-y plane letting s and α be polar coordinates and θ the angle between the two incident velocity vectors. In these coordinates it is readily shown that the energy transferred in a Coulomb collision is given by

$$\Delta E=\frac{E_1-E_2-m^2v_1v_2V(v_1\cos\theta-v_2)(s/2e^2)\cos\alpha\sin\theta}{1+(m/2e^2)^2V^2w^2s^2}, \quad (5)$$

where

$$w\equiv[(v_1\cos\theta-v_2)^2+v_1^2\sin^2\theta\sin^2\alpha]^{1/2}. \quad (6)$$

From Eq. (4) the cross section becomes

$$\frac{d\sigma(\mathbf{v}_1,\mathbf{v}_2)}{d(\Delta E)}=\frac{|v_2-v_1\cos\theta|}{V}\int d\alpha s\left|\frac{d(\Delta E)}{ds}\right|^{-1}. \quad (7)$$

Using Eq. (5) to find $d(\Delta E)/dS$ and $s(\alpha,\Delta E)$, and integrating over α, we find

$$\frac{d\sigma(\mathbf{v}_1,\mathbf{v}_2)}{d(\Delta E)}=\frac{4\pi e^4}{m^2V^4|\Delta E|}\left(\frac{E_1-E_2}{\Delta E}+\frac{2E_1E_2\sin^2\theta}{\Delta E^2}\right)$$
$$\text{for } |2\Delta E+E_2-E_1|\leq[(E_2-E_1)^2+4E_1E_2\sin^2\theta]^{1/2},$$
$$=0,\quad\text{otherwise}. \quad (8)$$

This result is essentially equivalent to the cross section found by Gryzinski.[6] It gives the total cross section for collisions with transfer of energy ΔE in the scattering of two beams of singly charged particles of mass m with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ ($\cos\theta\equiv\hat{v}_1\cdot\hat{v}_2$). It should be noted that Eq. (8) is completely symmetric with respect to the two particles. Asymmetrical cross sections for the scattering of either beam are related to Eq. (8) by

$$v_2d\sigma_2(\mathbf{v}_1,\mathbf{v}_2)=v_1d\sigma_1(\mathbf{v}_1,\mathbf{v}_2)=Vd\sigma(\mathbf{v}_1,\mathbf{v}_2)$$
$$=Nd(\Delta E)/N_1N_2, \quad (9)$$

where N is the total number of collisions per unit volume per unit time with transfer of energy ΔE, and N_1 and N_2 are the number densities of particles 1 and 2. For an isotropic velocity distribution for either particle we have

$$\frac{Vd\sigma(v_1,v_2)}{d(\Delta E)}=\frac{1}{4\pi}\int\frac{Vd\sigma(\mathbf{v}_1,\mathbf{v}_2)}{d(\Delta E)}2\pi\sin\theta d\theta. \quad (10)$$

Here the integration over angles must be confined to the region for which the condition given in Eq. (8) holds. This will include the whole range of θ, $0\leq\theta\leq\pi$ if

$$E_1E_2\leq(E_1-\Delta E)(E_2+\Delta E)\equiv E_1'E_2'. \quad (11)$$

If $E_1E_2\geq E_1'E_2'$ the limits on θ are given by

$$\cos^2\theta\leq E_1'E_2'/E_1E_2. \quad (12)$$

The integration of Eq. (10) may be carried out exactly. For $E_1E_2\leq E_1'E_2'$ it yields

$$\frac{d\sigma(v_1,v_2)}{d(\Delta E)}=\frac{\pi e^4}{(2mE_1E_2)^{1/2}|\Delta E|^3}\times\{|\Delta E|(E_1^{1/2}+E_2^{1/2}-|E_1^{1/2}-E_2^{1/2}|)+\tfrac{4}{3}(E_1^{3/2}+E_2^{3/2}-|E_1^{3/2}-E_2^{3/2}|)\}, \quad (13)$$

while in the case $E_1E_2\geq E_1'E_2'$ it yields

$$\frac{Vd\sigma(v_1,v_2)}{d(\Delta E)}=\frac{\pi e^4}{(2mE_1E_2)^{1/2}|\Delta E|^3}\times\{|\Delta E|(E_1'^{1/2}+E_2'^{1/2}-|E_1'^{1/2}-E_2'^{1/2}|)+\tfrac{4}{3}(E_1'^{3/2}+E_2'^{3/2}-|E_1'^{3/2}-E_2'^{3/2}|)\}. \quad (14)$$

Equations (13) and (14) may be combined into the more compact form

$$\frac{Vd\sigma(v_1,v_2)}{d(\Delta E)}=\frac{\pi e^4}{|\Delta E|^2}\left(\frac{2\mathcal{E}}{mE_1E_2}\right)^{1/2}\left[1+\frac{4}{3}\frac{\mathcal{E}}{|\Delta E|}\right], \quad (15)$$

where

$$\mathcal{E}\equiv[E_1,E_2,E_1',E_2']_< \quad (16)$$

is the smallest of the four ingoing and outcoming kinetic energies.[7]

IONIZATION CROSS SECTIONS

To find the ionization cross section we integrate Eq. (15) over $-U\geq\Delta E\geq-E_2$, where U is the ionization

[6] Our Eq. (8) is the same as Eq. (10) of Ref. 2 when the latter result is multiplied by $\Delta E/|\Delta E|$ as it should be. The equivalent to the quantity $f(\theta)d\theta$ of Ref. 2 is $d\Omega_{12}/4\pi$ in our treatment; hence the apparent discrepancy of a factor of 2 is not real.

[7] Equation (15) yields the exact cross section for binary Coulomb collisions. By replacing V by $(v_1^2+v_2^2)^{1/2}$ before integration over θ, Gryzinski (Ref. 2) obtains an approximate form of our Eq. (15) for the case $\Delta E<0$.

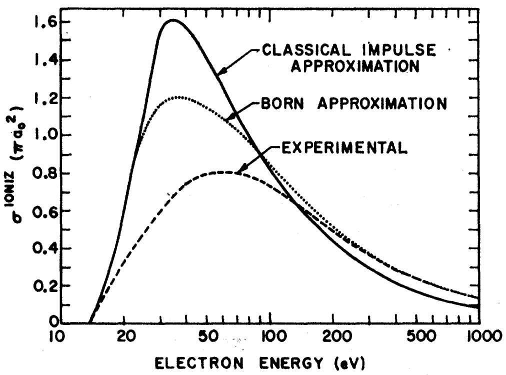

FIG. 2. Cross sections for the ionization of hydrogen obtained from the classical impulse approximation, the Born approximation, and from the measurements of Fite and Brackmann.[10]

potential of the target electron. In so doing we ignore the effect of the nuclear field on both the incident and target electrons except insofar as it determines the initial kinetic energy of the target electron E_1 and its ionization potential. This approximation should be good primarily for collisions in which E_2, $|\Delta E|\gg E_1$—that is, for close binary collisions of the electrons at moderate to high energies ($E_2>100$ eV). Thus, for the effective ionization cross section, we have

$$\sigma^{\rm ioniz}(E_2)\equiv\frac{1}{v_2}\int_{-E_2}^{-U}\frac{Vd\sigma(v_1,v_2)}{d(\Delta E)}d(\Delta E)\,, \qquad (17)$$

which, upon integration, becomes

$$\sigma^{\rm ioniz}(E_2)=\frac{2\pi e^4}{3E_2U^2}\frac{(E_2-U)^{3/2}}{E_1^{1/2}}\,,\quad U\le E_2<E_1+U$$

$$=\frac{\pi e^4}{3E_2}\left[\frac{2E_1+3U}{U^2}-\frac{3}{E_2-E_1}\right],\qquad E_2\ge E_1+U\,. \qquad (18)$$

This result should be compared with the classical ionization formula obtained by Thomson[8]:

$$\sigma_{\rm Th}{}^{\rm ioniz}(E_2)=(\pi e^4/E_2)(1/U-1/E_2)\,, \qquad (19)$$

which is identical to Eq. (18) if E_1 is set equal to zero. If both $E_2\gg E_1$ and $U\gg E_1$ there is little difference between the cross sections. The latter condition is never valid, however; in fact, for most atomic electrons $E_1>U$. Taking account of the motion of the bound electrons is seen, then, to increase the ionization cross section by a factor of about two through most of the energy range. In the high-energy limit the Thomson cross section should be multiplied by the factor $(1+2E_1/3U)$. The behavior at threshold is also different. Setting $E_1=0$ yields a cross section which increases linearly with the excess energy above threshold, while for $E_1\neq 0$ it increases with the 3/2 power of the excess energy. While neither treatment can be expected to yield accurate results in this domain it is interesting that the exact classical impulse approximation duplicates the 3/2 power law of the Born approximation at threshold while the earlier classical theory[1] quite fortuitously gives the currently accepted linear behavior at threshold.[9]

It is worth noting the form of Eq. (18) for hydrogenic atoms. For ionization of an energy level $E_n\equiv{\rm Ry}/n^2$ containing a single electron, the cross section becomes

$$\sigma^{\rm ioniz}(X)=\frac{8\pi a_0^2n^5}{3}\frac{(X-1/n^2)^{3/2}}{X}\,,\quad 1/n^2\le X\le 2/n^2$$

$$=\frac{4\pi a_0^2n^2}{3}\frac{5X-8/n^2}{X(X-1/n^2)}\,,\quad X\ge 2/n^2\,, \qquad (20)$$

where $X\equiv E_2/{\rm Ry}$.

In Fig. 2 we show the classical cross section for ionization of hydrogen in the ground state, along with the Born approximation and the experimental data of Fite and Brackmann.[10] Up to about twice the threshold energy the classical and Born results agree. The classical cross section has a maximum value which is 4/3 of the Born approximation value, and, while the agreement is not bad for intermediate energies, it worsens above 300 eV. The best agreement, within 15%, between the classical theory and experimental data lies in the region 100–300 eV; the worst agreement is near 30 eV where it is too large by a factor of 2.5.

EXCITATION CROSS SECTIONS

The cross section for excitation of a state with *total* energy U_n relative to the initial bound state is readily found from Eqs. (2) and (18):

$$\sigma^{\rm exc}=\frac{2\pi e^4}{3E_2E_1^{1/2}}\frac{(E_2-U_n)^{3/2}}{U_n^2}\,,\quad U_n\le E_2\le U_{n+1} \qquad (21a)$$

$$=\frac{2\pi e^4}{3E_2E_1^{1/2}}\left[\frac{(E_2-U_n)^{3/2}}{U_n^2}-\frac{(E_2-U_{n+1})^{3/2}}{U_{n+1}^2}\right],\qquad U_{n+1}\le E_2\le E_1+U_n \qquad (21b)$$

$$=\frac{2\pi e^4}{3E_2}\left[\frac{2E_1+3U_n}{2U_n^2}-\frac{3}{2(E_2-E_1)}-\frac{(E_2-U_{n+1})^{3/2}}{E_1^{1/2}U_{n+1}^2}\right],\qquad E_1+U_n\le E_2\le E_1+U_{n+1} \qquad (21c)$$

[8] See Ref. 1, or more conveniently M. J. Seaton, in *Atomic and Molecular Processes*, edited by D. R. Bates (Academic Press Inc., New York, 1962), p. 374.

[9] See Ref. 8; also S. Geltman, Phys. Rev. **102**, 171 (1956).

[10] W. L. Fite and R. T. Brackmann, Phys. Rev. **112**, 1141 (1958). Not shown are several variations of the Born approximation; see S. Geltman, M. R. H. Rudge, and M. J. Seaton, Proc. Phys. Soc. (London) **81**, 375 (1963).

$$=\frac{2\pi e^4}{3E_2}\left(\frac{1}{U_n}-\frac{1}{U_{n+1}}\right)\left[E_1\left(\frac{1}{U_n}+\frac{1}{U_{n+1}}\right)+\frac{3}{2}\right],$$

$$E_2\geq E_1+U_{n+1}.^{11} \quad (21d)$$

In practice almost the whole incident energy spectrum is spanned by Eqs. (21b) and (21d) since $\Delta U\equiv U_{n+1}-U_n$ is usually small. The cross sections given depend only on the initial and final energies—not on the angular momenta—of the electrons. Implicit in the calculation is a sum over all final angular momenta allowed by the conservation laws and the limits on the energy transfer. It should be noted that this classical theory can in principle be extended to distinguish between various final l values[12] but it becomes quite cumbersome.

In Fig. 3 we show the cross section for excitation of the $n=2$ levels of hydrogen as given by Eq. (21) as a function of the incident electron energy E_2. Also shown are the experimental results of Fite and Brackmann,[13] the Born approximation,[14] and the distorted wave calculation of Khashaba and Massey.[15] It is seen that the classical cross section is too peaked at its maximum value—a shortcoming which arises from its too rapid falloff at high incident energy [like $1/E$ rather than $(1/E)\log E$], and from its too slow rise at threshold

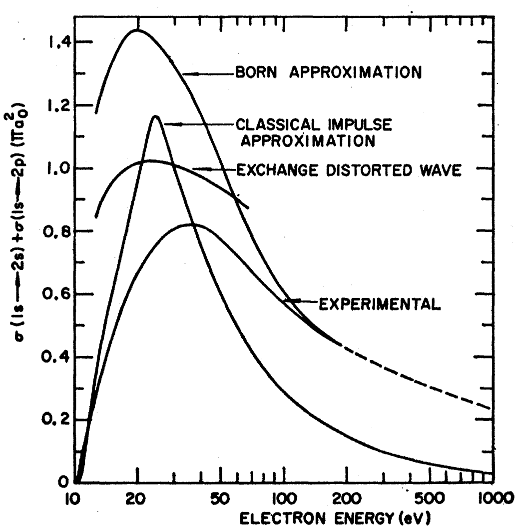

FIG. 3. Total cross sections for excitation of the $n=2$ ($l=1$ or 0) state of hydrogen obtained from the Born approximation,[14] the exchange distorted wave approximation,[15] the classical impulse approximation, and from the measurements of Fite and Brackmann.[13]

[11] It is assumed that $E_1\geq U_{n+1}-U_n$. For $E_1<U_{n+1}-U_n$ use Eq. (21c), omitting the third term, and Eq. (21d).
[12] C.f., Ref. 8, p. 378.
[13] W. L. Fite and R. T. Brackmann, Phys. Rev. **112**, 1151 (1958).
[14] V. M. Burke and M. J. Seaton, Monthly Notices Roy. Astron. Soc. **120**, 121 (1960). Not shown are the exchange (B.O.) or B II approximations, see P. G. Burke and K. Smith, Rev. Mod. Phys. **34**, 458 (1962).
[15] S. Khashaba and H. S. W. Massey, Proc. Phys. Soc. (London) **71**, 574 (1958).

[as $(E_2-U_n)^{3/2}$ rather than $(E_2-U_n)^{1/2}$]. Below 80 eV, however, the order of magnitude of the result is as good as that obtained from even second-order perturbation theory.

In Fig. 4 we show the cross section for excitation of the $n=4$ level of hydrogen from the $3d$ level as given by Eq. (21) and also as given by the Born approximation.[16] No experimental values are available, although it can be expected that the comparison would be similar to that shown in Fig. 3 for excitation of the $n=2$ level.

For higher states it is worthwhile expanding Eq. (21d) in powers of $1/n$. For a transition $n\to n+1$,

$$\sigma^{\text{exc}}(n\to n+1)\sim\frac{\pi a_0^2 n^4}{2}\left(\frac{\text{Ry}}{E_2}\right)\times\left(1+\frac{4}{n}-\frac{5}{3n^3}+\frac{4}{n^4}+\cdots\right),\quad E_2>E_n. \quad (22)$$

This equation again demonstrates a significant difference between the classical impulse approximation with and without allowance for the motion of the bound electrons. If we set $E_1=0$ as in the earlier classical theory the excitation cross section is

$$\begin{aligned}\sigma_{\text{Th}}^{\text{exc}}&=(\pi e^4/E_2)(1/U_n-1/E_2), &U_n\leq E_2\leq U_{n+1}\\ &=(\pi e^4/E_2)(1/U_n-1/U_{n+1}), &E_2\geq U_{n+1}.\end{aligned} \quad (23)$$

For a transition $n\to n+1$ where n is large this becomes

$$\sigma_{\text{Th}}^{\text{exc}}(n\to n+1)\sim\pi a_0^2 n^3\left(\frac{\text{Ry}}{E_2}\right)\times\left(1-\frac{1}{2n^2}+\frac{3}{4n^3}+\cdots\right),\quad E_2\geq\frac{4E_n}{n}. \quad (24)$$

Thus this cross section obtained with $E_1=0$ is smaller by a factor of $n/2$ than that found in Eq. (22) with $E_1=E_n$. As in the case of ionization the cross section is considerably enhanced by including the motion of the atomic electrons in the calculation. The n^4 dependence is more in accord with experiment and the Born approximation than the n^3 dependence.

Our ionization cross section given by Eq. (20) should be compared to our total cross section for excitation to all levels $n'>n$. In the limit of large n, the latter cross section is

$$\sigma_{\text{tot}}^{\text{exc}}=\frac{2\pi a_0^2 n^4}{3}\left(\frac{\text{Ry}}{E_2}\right),\quad n\gg1,\quad E_2>E_n, \quad (25)$$

which is $n^2/10$ times the corresponding ionization cross section obtained from Eq. (20).

[16] G. C. McCoyd, S. N. Milford, and J. J. Wahl, Phys. Rev. **119**, 149 (1960).

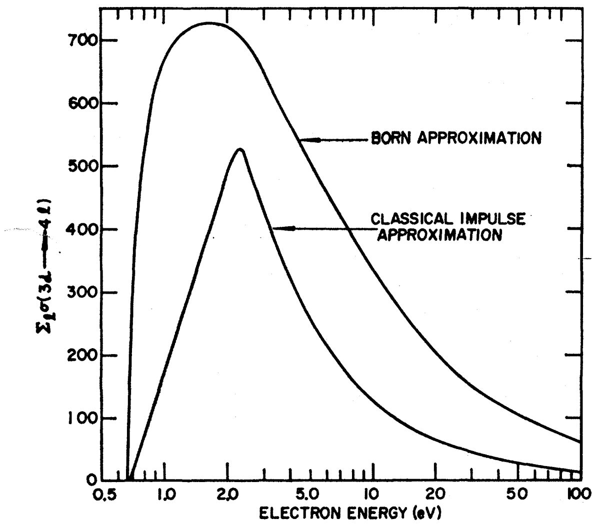

FIG. 4. Total cross sections for excitation of the $n=4$ ($l=0$, 1, 2, or 3) state of hydrogen obtained from the Born approximation[16] and the classical impulse approximation.

Also, in the limit of large n, excitation of the level n of hydrogen from the ground state is given by

$$\sigma^{\mathrm{exc}}(1\rightarrow n)\sim\frac{56\pi a_0^3}{3n^3}\left(\frac{\mathrm{Ry}}{E_2}\right),\quad E_2>2\ \mathrm{Ry}. \qquad (26)$$

The n^{-3} dependence for these transitions coincides with that for the squares of the dipole matrix elements in quantum theory for $n\gg1$ [17]—a result which points out the similarity between the Born approximation and this classical one.

CONCLUSIONS

Our motive for performing the above calculations has been more to clarify the predictions and point out the shortcomings of the classical impulse approximation than to suggest that it is an accurate way to obtain cross sections for various inelastic electron-atom collisions. Due to the lack of any theoretical or experimental values for most inelastic cross sections, the results of Gryzinski[2] have been applied extensively in calculation of electron-ion recombination coefficients,[18] and have recently been proposed[19] as the basis for further *semi*classical calculations of inelastic cross sections. They will probably continue to be used until such time as the results of better approximate quantal calculations are available.

We have shown here that the classical impulse approximation can be expected to fail at threshold and in the high-energy limit. It also cannot predict resonance effects. Between two and ten times the threshold energy, however, these cross sections for excitation or ionization are probably accurate to within a factor of about two. The other merit of this classical theory is that it provides the only analytic estimates, which also allow for differences in binding energies, for inelastic electron-atom cross sections. Even the Born approximation must be calculated by numerical methods and yields cross sections which are significantly better only in the high-energy limit.

A number of modification of the classical impulse approximation are possible. We consider Gryzinski's results[2,7] to be a modification of the above formulas which will in general improve the agreement with experiment due to the decreased weighting given in the total cross section to collisions with long interaction times. Of course a large number of similar modifications are possible which will also yield better agreement. The most physically meaningful of these is to choose the initial energy distribution of the target electron to be given by $p^2|\psi(p)|^2dp$, where $\psi(p)$ is the Fourier transform of the wave function of the target electron (as in the quantum impulse approximation[3]), rather than by the expectation value of the kinetic energy. Along this line Gryzinski[20] has noted that a continuous velocity distribution may yield the correct $E^{-1}\log E$ behavior for the classical ionization cross section at high energies.

Burgess[19] has obtained the correct high-energy and threshold behavior for ionization by treating distant collisions by the impact parameter method, the close collisions classically, and including exchange effects. There are other extensions of the classical approach for which the cross sections obtained here may be of some use. Finally we note that Eq. (15) may be applied in a straightforward way to find the rate of thermalization of charged particles as well as to find the cross sections for inelastic collisions.

Note added in proof. Some of the results obtained here have been found also by V. I. Ochkur and A. M. Petrun'kin.[21]

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